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# A Formal Concept Analysis Method for Path Planning Based on Formal Contexts

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**Abstract:** Path planning is widely applied in various fields, including robotic movement and 3D printing. In this paper, we propose a path planning method by combining formal concept analysis and the Q-learning algorithm to study grid map path planning. First, a grid map is represented as a formal context, and based on formal concept analysis theory, the necessary region concepts are defined and introduced into the Q-learning algorithm to obtain the optimal path. Finally, through system experiments, it has been proven that using the necessary region concept can reduce redundant information and the effectiveness of this method. The experimental results demonstrate that this method can identify an optimal path with fewer inflection points. This study may provide a new way to achieve optimal path planning based on formal concept analysis.

**Keywords:** Path planning, Formal concept analysis, Q-learning algorithm.

## 1. Introduction

Path planning is the process of determining the optimal path from a starting point to a target point through calculation and analysis [1-3]. This technology was originally applied in navigation and map services, but now it has been widely applied in various fields, such as robot navigation, autonomous driving, 3D printing [4], etc. The main purpose of path planning is to find the optimal path under given constraints with the minimum cost or to achieve a certain optimal goal. These constraints may include factors such as time, distance, and accessibility. By implementing path planning, efficiency can be enhanced, costs can be reduced, and the overall performance of systems can be improved. Path planning has seen significant developments, such as A* [5, 6], D*, and RRT [7, 8], and DWA [9, 10], TEB [11, 12]. In addition, the use of reinforcement learning algorithms [13, 14] to solve path planning problems has become a trending area of research.

Path planning is a crucial area of research within artificial intelligence and robotics. Regarding path planning algorithms, Fransen et al. [15] proposed an improved heuristic A algorithm for finding the lowest cost path in geometric graphs considering turning costs. Zhang et al. [16] proposed the penetration path planning of stealth drones in the integrated air defense system (IADS), in response to complex terrain and static radar threats, an improved A-Star algorithm including bidirectional sector expansion and variable step size search strategies is proposed. In this way, combined with the minimum radar cross-section (RCS) strategy, dynamic path planning is achieved. Recently, researchers have increasingly turned to deep learning and reinforcement learning to tackle path planning problems, leveraging the advancements in machine learning. Yan et al. [17] suggested a deep reinforcement learning (DRL) technique for unmanned aerial vehicle path planning that relies on global situational data. The simulation environment is provided using STAGE Scenario software, and a situational assessment model is developed to consider the survival probability of unmanned aerial vehicles against enemy radar detection and missile attacks.

For grid map path planning problems, the smallest unit of activity is each grid. There has been little research on the correlation between grids. Although A* algorithm and Dijkstra algorithm have some advantages in finding the shortest path, they tend to ignore the uncertainties on the path. Moreover, they are inefficient in grid maps with a large number of grids. Concept lattices are structures used to represent attributes shared by a group of objects, emphasizing the grid-to-grid relationship. Meanwhile, concept lattice theory is highly interpretable and can compensate for the lack of interpretability in path planning algorithms.

Formal concept analysis is a mathematical theory proposed by Wille [18] based on the binary relationship between objects and attributes in information systems. It is an important method in the field of granular computing [19, 20]. Based on this theory, many scholars have further researched and proposed there-way concept analysis [21, 22], fuzzy concept analysis [23], incomplete formal contexts, etc. Currently, formal concept analysis is used in granule description [24, 25], conflict analysis [26, 27], etc. Research in formal concept analysis is beneficial for enhancing the interpretability of applications. Wei et al. [28] proposed new concept lattices to support three-way decisions, defined two kinds of three-way operators and analyzed their properties, and proposed two types of three-way concept lattices. Xu et al. [29] proposed a new two-way CCL (TCCL) method in fuzzy contexts, aiming to solve the problems in existing two-way concept learning (TCL) and concept-cognitive learning (CCL), provide more flexible and efficient granular concept learning, and design a fuzzy-based progressive learning mechanism in the dynamic environment. Since formal concepts can have significant effects in other applications, it is best to try using them for path planning. Zhang et al. [30] proposed path planning based on formal contexts. They transformed formal concepts from formal contexts into formal sub-concepts and obtained region concepts. Based on the connectivity between region concepts connecting the starting and ending regions, they were able to find optimal paths in classical formal contexts. The research has achieved significant results in the application of practical path planning.

However, there are still some unsolved and important problems for this topic.

Existing path planning methods usually ignore the connections between grids when traversing grid maps. As a consequence, too many redundant grids are visited, which decreases the efficiency of path planning. Therefore, how to reduce redundant information in path planning should be seriously investigated.

The existing methods, such as the Q-learning algorithm, are not founded on concept cognitive learning theory and thus lack interpretability to some extent.

To this end, our research introduces formal concept analysis into path planning. Concretely, in order to enlarge the granularity of path selecting units and gain higher efficiency, we characterize the relationships among grids by necessary region concepts. Moreover, we adopt formal concept analysis by incorporating the spirit of the Q-learning algorithm.

The remaining parts are as follows. Section 2 introduces the relevant notions of formal concept analysis and path planning. Section 3 presents the Q-learning algorithm of reinforcement learning and optimization strategies for region concepts, use this algorithm to find the optimal path from the necessary region concepts. Section 4 introduces the conclusion and upcoming research.

## 2. Related Theoretical Foundations

The robot's activity area is represented as a grid map, which consists of both accessible areas and obstacle areas. By denoting accessible areas as 1, and obstacle areas as 0, the grid map can be interpreted as a formal context. Furthermore, a constraint is imposed on the transformed formal context that the order between rows or columns cannot be swapped.

Before proceeding, we recall some preliminaries of formal concept analysis. Formal concept analysis begins with a formal context, denoted as K= (O, A, I), where O is a set of objects, A is a set of attributes, and I represents the relationship between O and A. For $x \in O$, $y \in A$, $I(x, y) = 1$ means that the object $x$ possesses the attribute $y$, and $I(x, y) = 0$ means that the object $x$ does not possess the attribute $y$.

Example 1. A grid map is shown in Fig. 1. By denoting the accessible areas as 1 and the obstacle areas as 0, the grid map can be represented by K in Table 1.

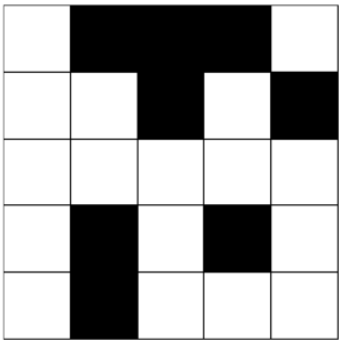

**Figure 1.** Grid map of Example 1

**Table 1.** The formal context K of Example 1

| | $y_0$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ |
|---|---|---|---|---|---|
| $x_0$ | 1 | 0 | 0 | 0 | 1 |
| $x_1$ | 1 | 1 | 0 | 1 | 0 |
| $x_2$ | 1 | 1 | 1 | 1 | 1 |
| $x_3$ | 1 | 0 | 1 | 0 | 1 |
| $x_4$ | 1 | 0 | 1 | 1 | 1 |

To build a formal concept, a pair of operators are defined as follows:

$$X^* = \{y \in A \mid \forall x \in X, I(x, y) = 1\}. \quad (1)$$

$$Y^* = \{x \in O \mid \forall y \in Y, I(x, y) = 1\}. \quad (2)$$

Moreover, if $X^* = Y$ and $Y^* = X$, then $(X, Y)$ is a formal concept, and $X$ and $Y$ are the extent and the intent of $(X, Y)$, respectively.

A type of partial order is defined as follows:

$$(X_1, Y_1) \le (X_2, Y_2) \Leftrightarrow X_1 \subseteq X_2 \Leftrightarrow Y_1 \supseteq Y_2. \quad (3)$$

In this way, all concepts can construct a complete lattice, denoted by L(K).

In what follows, we introduce formal concept analysis into path planning.

Let $X \subseteq O$. If $X = \{x_i, x_{i+1}, \cdots, x_{i+n}\}$, then we call $X$ is a continuous set of $O$. For instance, let $O = \{x_0, x_1, x_2, x_3, x_4\}$, then $\{x_2, x_3, x_4\}$, $\{x_0, x_1, x_2, x_3\}$ are two continuous sets of $O$. Moreover, if $X$ is a continuous set of $O$, and there does not exist another continuous set $X' \subseteq O$ such that $X \subset X'$, then we call $X$ a maximal continuous set of $O$.

Definition 1. Let $K = (O, A, I)$ be a formal context, and $(X, Y)$ be a formal concept. If $U \subseteq X$ and $V \subseteq Y$ are two maximal continuous sets, then we call $(U, V)$ a region concept, $rc$ for short.

Definition 2. Let $(X_1, Y_1)$ and $(X_2, Y_2)$ be two region concepts. If $X_1 \cap X_2 = X_I \ne \varnothing$ and $Y_1 \cap Y_2 = Y_I \ne \varnothing$, then it is said that the region concept $(X_1, Y_1)$ is connected to $(X_2, Y_2)$, and $(x_i, y_i)$ is called an inflection point, where $x_i \in X_I$ and $y_i \in Y_I$. Otherwise, it is said that $(X_1, Y_1)$ is not connected to $(X_2, Y_2)$.

Example 2. The concept lattice $L(K)$ is shown in Fig. 2, and region concepts are shown in Table 2. $(\{1, 2\}, \{1, 2\})$ and $(\{4\}, \{1, 2\})$ are two region concepts which are obtained from a formal concept $(\{1, 2, 4\}, \{1, 2\})$. In Table 2, $(\{0, 1, 2, 3, 4\}, \{0\})$ and $(\{2\}, \{0, 1, 2, 3, 4\})$ are two region concepts, due to $\{0, 1, 2, 3, 4\} \cap \{2\} = \{2\} \ne \varnothing$,

$\{0\}\cap\{0,1,2,3,4\}=\{0\}\neq\varnothing$ . Therefore, we call $(\{0,1,2,3,4\},\{0\})$ is connected to $(\{2\},\{0,1,2,3,4\})$, where the inflection point is $(2,0)$. In contrast, for the region concepts $(\{2,3,4\},\{0\})$ and $(\{1,2\},\{3\})$, while $\{2,3,4\}\cap\{1,2\}=\{2\}\neq\varnothing$ , $\{0\}\cap\{3\}=\varnothing$ . Thus, we call $(\{2,3,4\},\{0\})$ and $(\{1,2\},\{3\})$ are not connected.

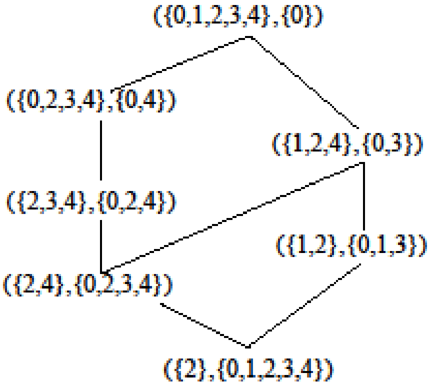

**Figure 2.** Concept lattice L(K) of Example 2

**Table 2.** Formal concepts and region concepts of Example 2

| Formal concepts | Region concepts |
|---|---|
| $(\{0,1,2,3,4\},\{0\})$ | $rc_{11}:(\{0,1,2,3,4\},\{0\})$ |
| $(\{0,2,3,4\},\{0,4\})$ | $rc_{21}:(\{0\},\{0\})$ |
| | $rc_{22}:(\{2,3,4\},\{0\})$ |
| | $rc_{23}:(\{0\},\{4\})$ |
| | $rc_{24}:(\{2,3,4\},\{4\})$ |
| $(\{1,2,4\},\{0,3\})$ | $rc_{31}:(\{1,2\},\{0\})$ |
| | $rc_{32}:(\{1,2\},\{3\})$ |
| | $rc_{33}:(\{4\},\{0\})$ |
| | $rc_{34}:(\{4\},\{3\})$ |
| $(\{2,3,4\},\{0,2,4\})$ | $rc_{41}:(\{2,3,4\},\{0\})$ |
| | $rc_{42}:(\{2,3,4\},\{2\})$ |
| | $rc_{43}:(\{2,3,4\},\{4\})$ |
| $(\{2,4\},\{0,2,3,4\})$ | $rc_{51}:(\{2\},\{0\})$ |
| | $rc_{52}:(\{4\},\{0\})$ |
| | $rc_{53}:(\{2\},\{2,3,4\})$ |
| | $rc_{54}:(\{4\},\{2,3,4\})$ |
| $(\{1,2\},\{0,1,3\})$ | $rc_{61}:(\{1,2\},\{0,1\})$ |
| | $rc_{62}:(\{1,2\},\{3\})$ |
| $(\{2\},\{0,1,2,3,4\})$ | $rc_{23}:(\{0\},\{4\})$ |

By Table 2, it can be observed that there are some identical region concepts induced by different formal concepts, such as $rc_{22}$ and $rc_{41}$. Besides, some larger region concepts contain some smaller ones; for example, $rc_{11}$ contains $rc_{21}$. It is trivial to show that these region concepts may contain redundant information, which should be eliminated in path planning.

## 3. Non-redundant Path Planning Based on Region Concepts

This section proposes a non-redundant path planning method based on region concepts. To improve the efficiency of the Q-learning algorithm, we remove unnecessary, duplicated, and irrelevant region concepts before performing path planning.

Proposition 1. Let $K=(O,A,I)$ be a formal context, $(X_1,Y_1)$ and $(X_2,Y_2)$ be two region concepts. If $X_1\subseteq X_2$ and $Y_1\subseteq Y_2$, then a path that visit $(X_1,Y_1)$ must visit $(X_2,Y_2)$.

Proof. Proof by contradiction. If a path visits $(X_1,Y_1)$ but does not visit $(X_2,Y_2)$, then it follows that there exists at least one point $(x,y)$ satisfy $x\in X_1$ , $y\in Y_1$ , and $x\notin X_2$ , $y\notin Y_2$, which is contrary to the condition that $X_1\subseteq X_2$ and $Y_1\subseteq Y_2$.

Proposition 1 actually states that using $(X_2,Y_2)$ for path planning yields the same result as using both $(X_1,Y_1)$ and $(X_2,Y_2)$ for path planning.

Definition 3. Let $K=(O,A,I)$ be a formal context, and $(X_1,Y_1)$ be a region concept. If there exists another region concept $(X_2,Y_2)$ such that $X_1\subseteq X_2$ and $Y_1\subseteq Y_2$, then $(X_1,Y_1)$ is called an useless region concept. Otherwise, $(X_2,Y_2)$ is called a necessary region concept, $nrc$ for short, all necessary region concepts are represented as $NRC$.

Definition 4. Let $(x_s,y_s)$ be a starting point, $(x_t,y_t)$ be a target point, and $(X,Y)$ be a necessary region concept. If $x_s\in X$ and $y_s\in Y$, then $(X,Y)$ is called a starting region concept, denoted as $rc_{start}$. If $x_t\in X$ and $y_t\in Y$, then $(X,Y)$ is called a target region concept, denoted as $rc_{target}$. In addition, the set of starting region concepts is denoted as $SRC_{start}$, and the set of target region concepts is denoted as $SRC_{target}$.

Example 3. Continued with Examples 1 and 2. Based on Definition 3, necessary region concepts and useless region concepts can be obtained. For instance, $(\{0\},\{0\})$ and $(\{0,1,2,3,4\},\{0\})$ are two region concepts, where $(\{0\},\{0\})$ is a useless region concept and $(\{0,1,2,3,4\},\{0\})$ is a necessary region concept. After removing the unless region concepts, the remaining ones are necessary region concepts, i.e., $(\{0\},\{4\})$ , $(\{2\},\{0,1,2,3,4\})$ , $(\{4\},\{2,3,4\})$ , $(\{1,2\},\{3\})$ , $(\{1,2\},\{0,1\})$, $(\{2,3,4\},\{2\})$, $(\{2,3,4\},\{4\})$ and $(\{0,1,2,3,4\},\{0\})$.

In Example 3, necessary region concepts account for 42% of the total number of region concepts. In addition, this study conducted comparative experiments by varying the proportion of obstacle areas.

To compare the number of necessary region concepts with the total number of region concepts, we conducted two group experiments based on five different formal contexts. Specifically, each group objects and 15 attributes, 20 objects and 20 attributes, 25 objects and 25 attributes, and 30 objects and 30 attributes. In the first experiment, the ratios of $(x_i, y_i) = 1$ and $(x_i, y_i) = 0$ were set at 8:2, 7:3, and 6:4, respectively. This indicates that obstacle areas constituted 20%, 30%, and 40% of the grid map, respectively. In the second experiment, the ratios were adjusted to 5:5, 4:6, and 3:7, with obstacle areas representing 50%, 60%, and 70% of the grid map.

The experimental results are shown in Fig. 3 and Fig. 4, respectively. The number of objects and attributes in the formal contexts corresponds to the size of the grid maps in the figures.

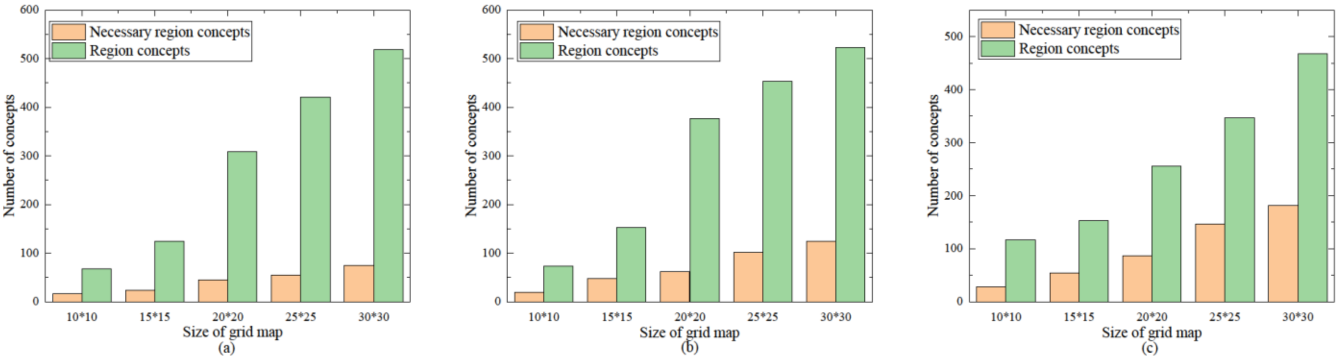

**Figure 3.** Number of region concepts in the first experiment

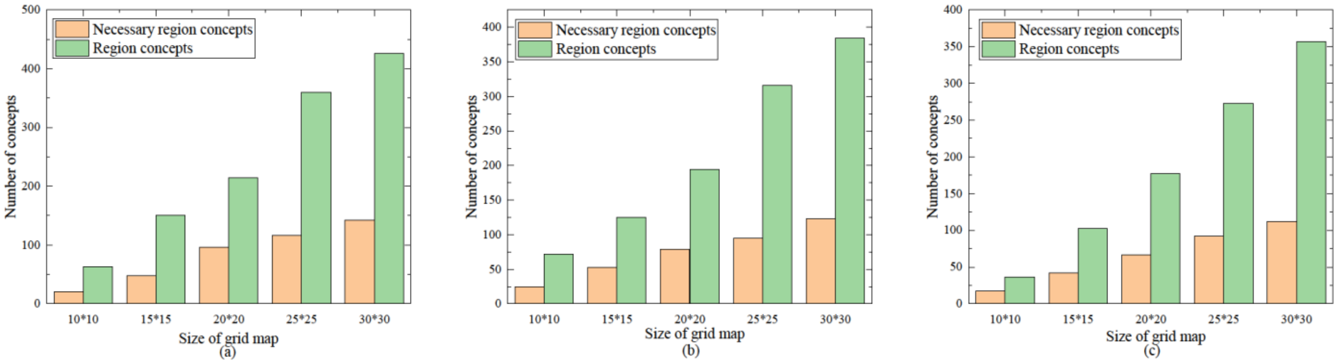

**Figure 4.** Number of region concepts in the second experiment

Based on the above experimental results, it is found that this optimization method can reduce the number of region concepts that need to be visited. In the first experiment, the necessary region concepts accounted for an average of 16.90%, 23.82%, and 34.88% of the total region concepts, respectively. In the second experiment, the necessary region concepts accounted for an average of 34.88%, 35.71%, and 38.26% of the total region concepts, respectively. By applying this optimization method, redundant region concepts are removed, leaving only the necessary region concepts. This reduction results in fewer necessary region concepts compared to the total number of grids and region concepts. As the efficiency of the Q-learning algorithm decreases with an increase in the number of nodes, applying this optimization method can improve the algorithm's efficiency.

To apply the Q-learning algorithm to path planning based on the necessary region concepts, it is essential to define the R-table and Q-table, which represent the reward value and the weight value, respectively.

The R-table is defined as a two-dimensional matrix, where each row represents a necessary region concept $nrc_s$, and each column represents a necessary region concept $nrc_a$. Each element $R(nrc_s, nrc_a)$ of the R-table corresponds to the immediate reward obtained from $nrc_s$ to $nrc_a$.

The Q-table is also defined as a two-dimensional matrix, where each row represents a necessary region concept $nrc_s$, and each column represents a necessary region concept $nrc_a$. Each element $Q(nrc_s, nrc_a)$ represents the weight from $nrc_s$ to $nrc_a$, which refers to the expected long-term cumulative reward from $nrc_s$ to $nrc_a$. Initially, the values in the Q-table are set to zero. Concretely, $Q(nrc_s, nrc_a) = 0$.

The Q-learning decision update rules for the Q-table during the algorithm execution are as follows:

$$Q(nrc_s, nrc_a) \leftarrow Q(nrc_s, nrc_a) + \alpha\left(R(nrc_s, nrc_a) + \gamma \max_{NRC_a'} Q(nrc_s', nrc_a') - Q(nrc_s, nrc_a)\right). \tag{4}$$

$nrc_s'$ is the next region concept of $nrc_s$, and $nrc_a'$ is the next region concept in this new region concept $nrc_s'$. $NRC_a'$ represents the set all connected $nrc_a'$ of $nrc_s'$. The learning rate, denoted by $\alpha$, $0 < \alpha < 1$, is used to reflect the step size of the update process. $\gamma$ represents the discount factor, $0<\gamma<1$, which is used to reflect the crucial future rewards.

The values of $\alpha$ and $\gamma$ influence the path weights but do not affect the optimal path, because these parameters primarily influence the overall weight of a given path without modifying the structural characteristics of the grid map. The optimal path is determined by the arrangement of accessible areas and obstacle areas, which define the feasible paths, and the grid map remains unchanged regardless of how $\alpha$ and $\gamma$ scale the path weights. In all experiments presented in this paper, we set $\alpha=0.4$ and $\gamma=0.3$, as this configuration provides the most distinction in path weights.

To obtain an optimal path, combining the Q-learning algorithm and necessary region concepts, we propose a non-redundant path planning method, as shown in Table 3.

**Table 3.** Non-redundant path planning method based on formal context

| |
|---|
| Required: A grid map M, a starting point, and a target point. |
| Ensure: An optimal path and average weight |
| Step 1: Construct a formal context K from the grid map M. |
| Step 2: Construct the concept lattice L(K) based on K. |
| Step 3: Derive the region concepts. |
| Step 4: Remove useless region concepts to obtain the necessary region concepts, $SRC_{start}$ and $SRC_{target}$. |
| Step 5: Calculate the reward values in the R-table based on the necessary region concepts. |
| Step6: Generate the Q-table using the Q-learning algorithm, based on the R-table. |
| Step 6: Extract the optimal path and the corresponding average weight from the Q-table. |

Based on $SRC_{start}$, $SRC_{target}$, and the R-table, the Q-table can be obtained by Algorithm 1. First, for any of $SRC_{start}$, select the connected necessary region concept with the highest reward value based on the R-table, execute Q-learning decision updates, and record the Q-learning decision result in the Q-table. Then, continue selecting the next connected necessary region concept with the highest reward value and execute Q-learning decision updates until a $rc_{target}$ is reached. Repeat this process a predetermined number of times to obtain the final Q-table.

In Algorithm 1, construct L(K) from the formal context K and derive necessary region concepts, then obtain the optimal path based on the connectivity between these necessary region concepts.

The average weight of path, denoted as AW, is defined as follows:

$$AW=\frac{\sum_{nrc_s\in NRC_{path}} Q(nrc_s,nrc_a)}{n-1}. \tag{5}$$

$n$ represents the number of necessary region concepts visited by the path. $Q(nrc_s,nrc_a)$ represents the weight value from $nrc_s$ to $nrc_a$. $NRC_{path}$ represents all the $nrc$ visited by the path. The path with the highest $AW$ among all paths obtained by Algorithm 1 is the optimal path.

Example 4. Continued with Example 3. Let $(0,0)$ be the starting point and $(4,4)$ be the target point. Table 3 show that $(\{0,1,2,3,4\},\{0\})$ is $rc_{start}$, while $(\{4\},\{2,3,4\})$ and $(\{2,3,4\},\{4\})$ are $rc_{target}$. In setting the values of the R-table, we set $R(nrc_s,nrc_a)=0$ when $nrc_s$ and $nrc_a$ are not connected, and $R(nrc_s,nrc_a)=20$ when they are connected. Specifically, when $nrc_a\in SRC_{target}$, we set $R(nrc_s,nrc_a)=100$. Through multiple experiments and parameter tuning, it was found that with this setting, the path can converge within a short period of time, leading to better solutions.

$rc_{start}$ is $(\{0,1,2,3,4\},\{0\})$, and the region concepts connected to $(\{0,1,2,3,4\},\{0\})$ are $(\{2\},\{0,1,2,3,4\})$ and $(\{1,2\},\{0,1\})$. In the Q-learning algorithm, the next region concept of $(\{0,1,2,3,4\},\{0\})$ will be $(\{2\},\{0,1,2,3,4\})$ or $(\{1,2\},\{0,1\})$.

Loop 1. When $(\{2\},\{0,1,2,3,4\})$ is chosen, then $Q((\{0,1,2,3,4\},\{0\}),(\{2\},\{0,1,2,3,4\}))=2$. Due to $(\{2,3,4\},\{4\})$ as $rc_{target}$, the next region concept of $(\{2\},\{0,1,2,3,4\})$ is $(\{2,3,4\},\{4\})$, with $Q((\{2\},\{0,1,2,3,4\}),(\{2,3,4\},\{4]))=10$. This results path is $(0,0)\rightarrow(2,0)\rightarrow(2,4)\rightarrow(4,4)$, completing the loop.

Loop 2. When $(\{1,2\},\{0,1\})$ is chosen, then $Q((\{0,1,2,3,4\},\{0\}),(\{1,2\},\{0,1\}))=2$. Then, the region concept connected to $(\{1,2\},\{0,1\})$ is $(\{2\},\{0,1,2,3,4\})$, $Q((\{1,2\},\{0,1\}),(\{2\},\{0,1,2,3,4\}))=2$. Finally, the next region concept of $(\{2\},\{0,1,2,3,4\})$ is $(\{2,3,4\},\{4\})$, with $Q((\{2\},\{0,1,2,3,4\}),(\{2,3,4\},\{4\}))=10$. This results path is $(0,0)\rightarrow(2,0)\rightarrow(2,4)\rightarrow(4,4)$, completing the loop.

$$R=\begin{pmatrix}0&0&0&0&0&0&0&0\\0&0&0&20&20&20&100&20\\0&0&0&0&0&20&100&0\\0&20&0&0&0&0&0&0\\0&20&0&0&0&0&0&20\\0&20&100&0&0&0&0&0\\0&20&100&0&0&0&0&0\\0&20&0&0&20&0&0&0\end{pmatrix}$$

Although the paths obtained from Loop 1 and Loop 2 are identical, Loop 2 involves an additional selection of necessary region concepts, which results in a lower AW for the path.

After executing the process 1000 times, the contents of the Q-table stabilized, indicating that the algorithm had converged and the path result had become stable. Ultimately, the final weight values in the Q-table were obtained.

$$Q=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23 & 23 & 23 & 100 & 23 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 & 0 & 0 & 0 & 23 \\ 0 & 30 & 100 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 & 23 & 0 & 0 & 0 \end{pmatrix}$$

According to the Q-table, $Q((\{0,1,2,3,4\},\{0\}),(\{2\},\{0,1,2,3,4\}))$ is higher than $Q((\{0,1,2,3,4\},\{0\}),(\{1,2\},\{0,1\}))$, so the next region concept of $(\{0,1,2,3,4\},\{0\})$ is $(\{2\},\{0,1,2,3,4\})$. $Q((\{2\},\{0,1,2,3,4\}),(\{2,3,4\},\{4\}))$ is highest among all $Q((\{2\},\{0,1,2,3,4\}),nrc_a)$, so the next region concept of $(\{2\},\{0,1,2,3,4\})$ is $(\{2,3,4\},\{4\})$. Due to $(\{2,3,4\},\{4\})$ being the target region concept, the final path obtained is found as $(0,0)\rightarrow(2,0)\rightarrow(2,4)\rightarrow(4,4)$. The inflection points are $(2,0)$ and $(2,4)$.

In addition, this paper is also an improvement on the Q-learning algorithm. Due to the robot's movable region being a grid map, where the smallest unit of activity is each grid, the classic Q-learning algorithm tends to generate very large R-tables and Q-tables. To address this issue, this study utilizes formal concept analysis to derive necessary region concepts and use the necessary region concepts as the smallest unit for activities. This approach reduces the number of the smallest units in the Q-learning algorithm, thereby decreasing the data complexity and improving efficiency.

# 4. Conclusions

This paper studies the path planning problem based on formal contexts and proposes a Non-redundant path planning method. Specifically, the necessary region concepts are used as the smallest unit for activities in the Q-Learning algorithm, which decreases the complexity of the Q-Learning algorithm when processing grid maps and obtains an optimal path with fewer inflection points.

This paper combines reinforcement learning to study path planning and can obtain an optimal path, which provides a new direction for the application of formal concept analysis. In the future, further exploration and research could focus on the integration of reinforcement learning and formal concept analysis.

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