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The Application of the Function and Equation Thought in Middle School Mathematics Problem-Solving

Xinke Deng *, Junpeng Yuan, Jun Liu, Lin Zhang, Fuqi You, Guangjin Liu, Yantong Li, Liangxiong Lu

School of Mathematics and Information Sciences, Guangzhou University, GuangZhou, 510006, China
#These authors contributed equally.
* **Corresponding author**: (Email: 1305422736@qq.com)

Abstract: The idea of functions and equations is a core approach running through algebra, geometry, probability and other knowledge modules in middle school mathematics. Its value in cross-module problem-solving and connection with curriculum standards has become increasingly prominent. Taking sequences, trigonometric functions and solid geometry as research objects, this study systematically analyzes the problem-solving logic of the function and equation idea, so as to provide empirical references for front-line teaching to implement curriculum standards and improve students' key mathematical competencies. This paper adopts literature review, case analysis and classification research. First, it defines the connotation and relationship between function thought and equation thought, clarifies their core feature of "interconversion", and meets the curriculum standard requirement of "understanding the essence of mathematical thought". Second, based on recent college entrance examination questions and typical examples, it analyzes the application paths of these thoughts by module. Finally, it summarizes their general problem-solving logic and puts forward teaching and problem-solving suggestions in line with curriculum standards. The research shows that the function and equation idea permeates middle school mathematics and is crucial for developing core competencies such as mathematical abstraction and logical reasoning. It is often tested with the combination of numbers and shapes, classified discussion in the college entrance examination, reflecting the orientation of "emphasizing literacy and application". It simplifies complex problems and improves students' problem-solving efficiency and thinking quality.

Keywords: Function and equation idea, Middle school mathematics, Core competencies, Curriculum standards, Problem-solving logic.

1. Introduction

The idea of functions and equations is a core approach in middle school mathematics. It runs through multiple knowledge modules such as algebra, geometry, and probability, and is an important vehicle for cultivating students' mathematical abstraction and logical reasoning literacy. With the advancement of the new curriculum reform, the college entrance examination questions have increasingly focused on the comprehensive examination of thinking methods.

In recent years, scholars have conducted extensive research on the application of function and equation thinking in middle school mathematics. Zhao Xue (2025) explored the application value of this thinking in high school mathematics problem-solving, pointing out its role in cultivating students' problem-solving thinking[1]; Zhang Caini (2025) investigated the unit teaching design of "functions and equations" from the perspective of core literacy, providing a practical pathway for the classroom infiltration of this thinking method[2]; Liu Bin et al. (2025) demonstrated the practical value of function and equation thinking across different knowledge modules through case analysis[3]; Ma Rui and Wen Bin (2025) focused on teaching strategies for mathematical thinking methods under the core literacy orientation, offering theoretical guidance for front-line teaching[4]; He Shiguang (2024) analyzed the specific application of this thinking in problem-solving through examples, emphasizing that teachers should apply it flexibly according to different problem types[5]; Ding Mingyi (2025) explored how to improve students' core competencies by grasping the essence of functions from a holistic perspective[6]. These studies provide an important theoretical foundation and basis for our research. Building upon this foundation, this paper aims to further systematically integrate the application of function and equation thinking across multiple knowledge modules.

To achieve this goal, this paper first clarifies the connotation and connection between function thinking and equation thinking, and then, in combination with the requirements of the new curriculum standards, systematically analyzes their application in solving problems in core middle school mathematics modules such as sequences, trigonometric functions, and solid geometry. The study focuses on recent college entrance examination questions and, through the dissection of typical examples, reveals the problem-solving logic of function and equation thinking, aiming to provide practical references for front-line teaching and help students improve their ability to apply core thinking methods to solve complex problems.

2. Thinking of Functions and Equations

2.1. Definition of Concepts

2.1.1. Function Thinking

Function thinking refers to the use of the perspective of motion and change, the intrinsic connection between sets and their correspondence to analyze and study quantitative relationships in mathematical problems, to establish functional relationships or construct functions, and to analyze, transform, and solve problems by studying the properties of functions, such as monotonicity, continuity, periodicity, extremum, etc. Function thinking is one of the most core and fundamental ways of thinking in mathematics, and its essence is to describe the dynamic associations in reality or mathematics through the "correspondence between variables".

2.1.2. The Idea of Equations

The idea of equations is the core idea in mathematics for solving unknowns and characterizing equal relationships. It transforms the textual relationships in the problem into mathematical symbolic language by establishing equalities (equations) containing unknowns, and then uses the properties of equalities and the methods for solving various types of equations to derive the values of unknowns. It is the key bridge connecting the known and the unknown.

2.2. The Connection Between the Function Idea and The Equation Idea

The idea of functions and the idea of equations, as two core ideas in the mathematical system, are not isolated from each other but present a dialectical unity of the particular and the general, the static and the dynamic. In essence, an equation is a special state of a function - any function is transformed into an equation when its value is defined as a fixed constant C, and the roots of the equation correspond to the values of the independent variables at the intersection of the function graph and the line y = f(x); f(x) = C; y = C ; A function is a general extension of an equation, and the expression of the function itself can be regarded as a binary implicit equation about andxy. Function relations can be regarded as equations, and some equations can be regarded as function relations. In solving certain problems, functions and equations often transform and permeate each other.

2.3. Requirements of the New Curriculum

Take the "General Senior High School Mathematics Curriculum Standards" as an example, the significance of the thinking methods of functions and equations can be reflected from it.

(1) The teaching requirements for derivatives of single-variable functions and their applications mention "being able to analyze the laws of change of things using the idea of functions and express the laws and construct models using the language of functions."

(2) Teaching quality Description Level One mentions "in familiar situations, be able to establish the concept of functions from the corresponding perspective, understand the background, concepts and properties of basic function classes (see academic requirements for the function topic), be able to explain the laws of change of things using properties such as monotonicity of functions, and have the awareness to analyze problems using the function thinking."

(3) Application of functions mentions "being able to select appropriate functions to construct mathematical models for simple practical problems and solve them; Be able to understand equations from the perspective of functions and use the properties of functions to find approximate solutions to equations."

(4) The concept of complex numbers mentions "recognizing complex numbers through the solution of the equation."

(5) In circles and Equations, it is mentioned that "further understand the idea of combining numbers and shapes through the study of conic sections and equations."

It is not difficult to see from the general High school mathematics curriculum standards that functions and equations, as the basic content of high school mathematics, are closely related to many knowledge points and are widely applied.

3. The Idea of Functions and Equations for Problem-Solving Applications

3.1. Sequences

A sequence is a particular function whose domain is a set of positive integers or a subset of them. In solving sequence problems, using the function perspective, the general term formula of the arithmetic sequence can be regarded as a linear function, and the sum formula of the first n terms as a quadratic function; The properties of functions can also be used to analyze the characteristics of sequences; It can also handle complex problems such as evaluating and summing sequences in combination with functions through constructors.

Example 1: Given the function $f(x) = (x-3)^3 + x - 1$, if the sequence $\{a_n\}$ is an arithmetic sequence with a non-zero common difference, and $f(a_1) + f(a_2) + \cdots + f(a_7) = 14$, then $a_1 + a_2 + \cdots + a_7 =$ ( )

A.0 B.7 C.14 D.21

Analysis: Given $f(x) = (x-3)^3 + x - 1$, we can derive $f(x) - 2 = (x-3)^3 + x - 3$. Construct the function $g(x) = f(x) - 2$, which is symmetric about the point (3,0). Using the condition $f(a_1) + f(a_2) + \cdots + f(a_7) = 14$, we obtain $g(a_1) + g(a_2) + \cdots + g(a_7) = 0$. Since g(x) is symmetric about (3,0) and the sum of the function values at 7 points is 0, the middle point g(a_4) must be the intersection of g(x) with the x-axis,a_4=3. From this, we can find the value of $a_1 + a_2 + \cdots + a_7$.

Solution: Since $f(x) = (x-3)^3 + x - 1$, then $f(x) - 2 = (x-3)^3 + x - 3$. Let $g(x)= f(x) -2$, so the graph of g(x) is symmetric about the point (3,0). Because $f(a_1) + f(a_2) + \cdots + f(a_7) = 14$, it follows that $[f(a_1) - 2] + [f(a_2) - 2] + \cdots + [f(a_7) - 2] = 0$, which means $g(a_1) + g(a_2) + \cdots g(a_7) = 0$. Since g(x) is symmetric about (3, 0) and the sum of the seven function values is 0, the middle term g(a_4) must be 0, so a_4= 3. Therefore, $a_1 + a_2 + \cdots + a_7 = 7a_4 = 7 \times 3 = 21$. Thus, the answer is D.

3.2. Trigonometric Functions

The idea of functions and equations is a key tool for solving trigonometric function problems. The idea of functions is reflected in treating trigonometric functions as basic elementary functions and analyzing problems by utilizing their monotonicity, periodicity, parity, extremum and other properties. The idea of equations is to solve problems by transforming them into equations or systems of equations through trigonometric identities and variable substitutions. The combination of the two can efficiently break through various types of trigonometric problems.

Example 2: (2024 National College Entrance Examination Paper I question 15) Mark the opposite sides of the interior angles A, B, and C of triangle ABC as a, b, and c respectively, given $\sin C = \sqrt{2}\cos B.a^2 + b^2 - c^2 = \sqrt{2}ab$

(1) Find B. (2) if the area of triangle ABC is $3 + \sqrt{3}$, find c.

Analysis: This is a comprehensive problem of solving triangles. It mainly examines the application of the idea of functions and equations in the transformation of the relationship between sides and angles, and requires the combination of the cosine theorem, the sine theorem, and the triangle area formula for solving. The key to question 1 is to use the idea of equations to transform the relationship of the sides into that of the angles. Question 2 also embodies the idea of equations, transforming the area and the relationship between the edges and corners into an equation about c.

Solution:

(1) According to the Law of Cosines, $a^2 + b^2 - c^2 = 2ab\cos C = \sqrt{2}ab$; Solving this gives $\cos C = \frac{\sqrt{2}}{2}$. Since $C \in (0, \pi)$, we have $C = \frac{\pi}{4}$.

From $\sin C = \sqrt{2}\cos B$, we get $\cos B = \frac{1}{2}$. Since $B \in (0, \pi)$, we solve for $B = \frac{\pi}{3}$.

(2) According to the Law of Sines, $\frac{b}{\sin B} = \frac{c}{\sin C}$, $b = \frac{c\sin B}{\sin C} = \frac{\sqrt{6}}{2}c$; Also, $\sin A = \sin(B + C) = \sin B\cos C + \cos B\sin C = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$.

Using the triangle area formula $S = \frac{1}{2}bc\sin A = 3 + \sqrt{3}$, we substitute: $\frac{1}{2} \cdot \frac{\sqrt{6}}{2}c \cdot c \cdot \frac{\sqrt{6}+\sqrt{2}}{4} = 3 + \sqrt{3}$. Solving this equation gives $c = 2\sqrt{2}$.

3.3. Solid Geometry

Example 3: As shown in the Figure 1, in trapezoid ABCD, AB is parallel to CD, BC is perpendicular to AB, with BC = CD = 2 and AB = 4. Fold triangle BCD along BD to form triangle BPD, as shown in Figure 2, where P is the moving point.

(1) When plane ABD intersects plane BPD,

(i) Verify that AD ⊥ BP

(ii) Calculate the distance from point B to plane ADP.

(2) Determine the maximum value of the sine of the angle formed between line AP and plane ABD.

As shown in Figures 1 and 2.

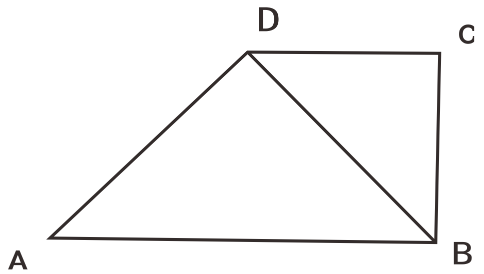

Figure 1. Plane Geometry Prototype Diagram (Before Folding)

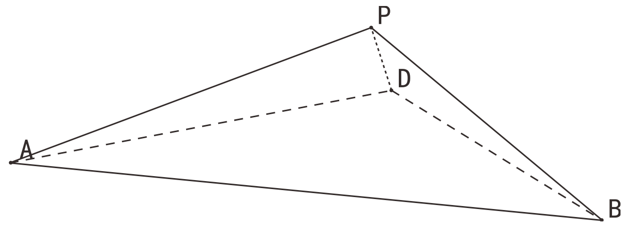

Figure 2. Intuitive Solid Geometry Figure After Folding

Analysis: First, determine the position of projection O of P on the plane ABD, then convert the sine value of the line-plane angle into the ratio of OP to AP. By setting OM = x to establish a functional relationship, and after substitution, utilize the fundamental inequality to find the maximum value of the function, thereby obtaining the maximum value of the sine of the line-plane angle.

Analysis:

(1) (i)In the trapezoid ABCD, since AB//CD and BC⊥AB, with BC=CD=2 and AB=4, AD=BD=$2\sqrt{2}$. Thus, $AD^2 + BD^2 = AB^2$. Consequently, AD⊥BD.

Given that plane ABD⊥plane BPD, the intersection of planes ABD and BPD is BD, and AD⊂plane ABD, AD⊥plane BPD. Since BP⊂plane BPD, so AD⊥BP.

(ii) Because AD⊥BP and BP⊥PD, AD∩PD=D. Since AD and PD⊂plane APD, so BP⊥plane APD. Hence, the distance from point B to plane APD is BP=2.

(2) Let M be the midpoint of BD and E the midpoint of AB. Connect EM and PM, where PM = 2.

According to (1), EM⊥BD and PM⊥BD.

Let O be the projection of point P onto plane ABD. Since PB = PD and PO = PO, we have RtΔPOB ≅ RtΔPOD, thus OB = OD, and O lies on line EM.

Let the angle between the line AP and the plane ABD be θ, then $\sin\theta = \frac{OP}{AP}$.

Clearly, when O is symmetrically positioned on either side of BD, OP has equal length. However, when O is on the right side of BD, AP is longer, resulting in a smaller sin θ = OP/AP. Therefore, we only need to consider the cases where O is on BD or to its left, as shown in Figure 3.

As shown in Figure 3.

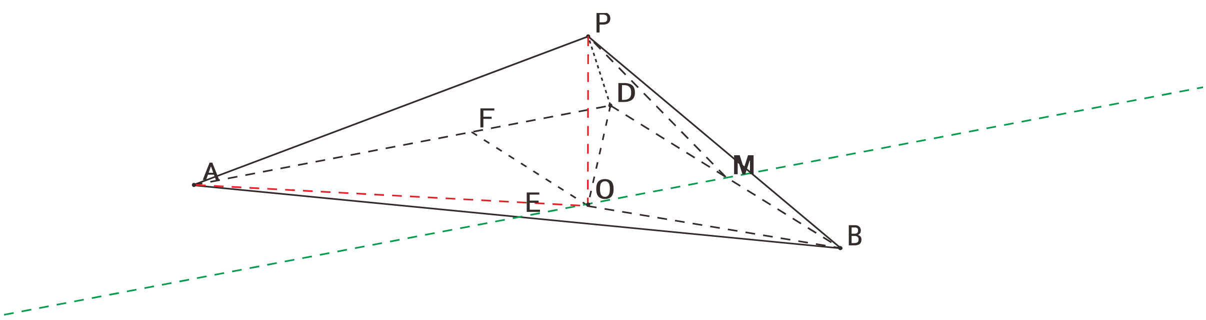

Figure 3. Diagram of Projection and Line-Plane Angle for Triangular Pyramid P-ABD

Draw $OF \perp AD$ through point O, then $OP = DM = \sqrt{2}$.

Let $OM = x \in [0, \sqrt{2})$, then $DF = x$, $AF = AD - DF = 2\sqrt{2} - x$.

$AO = \sqrt{AF^2 + OF^2} = \sqrt{x^2 - 4\sqrt{2}x + 10}$, $OP = \sqrt{PM^2 - OM^2} = \sqrt{2 - x^2}$. Thus, $AP = \sqrt{AO^2 + OP^2} = \sqrt{12 - 4\sqrt{2}x}$.

Therefore, $\sin\theta = \frac{OP}{AP} = \frac{\sqrt{2-x^2}}{\sqrt{12-4\sqrt{2}x}}$, $\sin^2\theta = \frac{1}{4}\cdot\frac{2-x^2}{3-\sqrt{2}x}$. Let $t = 3-\sqrt{2}x$, where $t\in(1,3]$. Then $x = \frac{3-t}{\sqrt{2}}$, and $\sin^2\theta = \frac{1}{8}\cdot\frac{-t^2+6t-5}{t} = \frac{1}{8}\left[6-\left(t+\frac{5}{t}\right)\right] \le \frac{3-\sqrt{5}}{4}$.

The equality holds if and only if $t=\sqrt{5}$, i.e.$x = \frac{3\sqrt{2}-\sqrt{10}}{2}$. Therefore, $\sin\theta = \frac{\sqrt{10}-\sqrt{2}}{4}$. That is, the maximum value of the sine of the angle formed by the line AP with the plane ABD is $\sin\theta = \frac{\sqrt{10}-\sqrt{2}}{4}$

4. Conclusion

4.1. Research Summary

The middle school stage is the golden period for students' thought learning in the corresponding subject, and mathematical thought learning during this stage requires particular attention from teachers and students. This paper focuses on the core ideas and methods of middle school mathematics - the idea of functions and equations. It first systematically defines the connotations and intrinsic connections of the two ideas in the second part, and then closely follows the requirements for cultivating mathematical core literacy in the new curriculum standards in the third part, combined with recent college entrance examination questions and typical examples, A comprehensive analysis of their application in solving problems in three modules: sequences, trigonometric functions, and solid geometry.

In different knowledge modules, the application of these two ideas shows a distinct emphasis. The study, through the dissection and logical deduction of example problems in each module, found that thought methods play an important role in learning mathematics, especially in solving mathematical problems. Once the mathematical ideas of functions and equations are mastered, many problems can be easily solved. At the same time, the college entrance examination questions and typical examples also reveal that the idea of functions and equations is often integrated with ideas such as the combination of numbers and shapes, classification discussion, and transformation, emphasizing the comprehensive transfer and practical application of knowledge, reflecting the proposition principle of "emphasizing literacy and application" of the new curriculum standards[7], and putting forward higher requirements for students' logical reasoning ability. It provides targeted references for the practice of middle school mathematics teaching and the improvement of students' problem-solving abilities.

Based on the above research conclusions, the researchers put forward the following suggestions for teaching and learning in order to enhance the effectiveness of the idea of functions and equations in teaching and problem-solving:

First of all, how to cultivate students' mathematical thinking and mathematical thinking quality in the middle school teaching process is also one of the issues that front-line teachers and researchers should pay attention to, so there are two suggestions for teachers:

(1) Teachers should dig deep into the implicit carriers of ideas and methods in the teaching materials, design stratified teaching cases in combination with the college entrance examination questions, and help students build a knowledge network for the application of ideas through the process of "example dissection - method refinement - variant training";

(2) Focus on the cultivation of core competencies and guide students to understand the logical essence behind the thinking methods rather than mechanically memorizing the problem-solving steps.

Secondly, for students, it is necessary to strengthen the core awareness of "transformation and conversion", actively construct functional relationships or equation models in problem-solving, and accumulate typical application scenarios of different modules.

4.2. Prospects

This study presents the application of the idea of functions and equations in middle school mathematics problem-solving through specific examples, providing reference suggestions for students to apply this idea in problem-solving. The idea of functions and equations is an important carrier for the cultivation of core ideas, methods and literacy in middle school mathematics. This study only conducts a preliminary exploration of its application in problem-solving, and subsequent research can be deepened in the following aspects:

First, construct an integrated teaching practice path of "thought infiltration - literacy cultivation - interest stimulation". Based on the actual classroom teaching, design life-situation and cross-disciplinary tasks, embed the application of thought methods into real problem-solving, form a practical framework for thinking cultivation, and enhance students' interest through practical exploration.

Secondly, conduct empirical research and effect evaluation by educational stage and level. Design stratified research tools and test cases based on the knowledge base of junior high school and senior high school, as well as the ability differences among students of different academic levels, analyze the application difficulties and cognitive patterns of each group, develop teaching resources suitable for different school stages and levels, and improve the teaching adaptability of research.

Thirdly, deepen the research on the correlation between ideological methods and literacy under the guidance of the new curriculum standards. Further interpret the core literacy content of the new curriculum standards, and design corresponding question types based on emerging proposition directions such as interdisciplinary integration and real-life situations, so that the research is more in line with the current teaching needs for literacy cultivation.

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