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A Review of State Estimation for Time-Delay Neural Networks

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Abstract: In practical engineering, neural networks inevitably suffer from time-delay phenomena caused by signal transmission, hardware response, and data communication. Time-delay will degrade the dynamic performance of neural networks, even lead to oscillation, chaos, and instability. Meanwhile, affected by sensor constraints and external interference, only a small number of system states can be measured directly, which makes state estimation become one of the key technologies in the analysis and application of time-delay neural networks. State estimation refers to reconstructing the real full state of the system through measurable output data, which provides the state basis for stability analysis, fault detection, and feedback control. In recent years, state estimation of time-delay neural networks has become a research hotspot in the fields of control science, intelligent systems, and dynamical systems. This paper systematically summarizes the research results of state estimation for time-delay neural networks. Firstly, the basic model, delay types, and activation function constraints are introduced. Secondly, the core theoretical tools including Lyapunov-Krasovskii functional method, various integral inequalities, reciprocally convex combination lemmas, S-Procedure, and switched system methods are summarized. Thirdly, the research progress of $H\infty$ state estimation, L_2-$L\infty$ state estimation, dissipative state estimation, and event-triggered state estimation is reviewed in detail. Finally, the existing problems and challenges in this field are summarized, and the future development directions including low conservatism, intelligence, network security, and practical application are discussed.

Keywords: Time-delay neural networks, state estimation, Lyapunov-Krasovskii functional, integral inequality, switched system, event-triggered mechanism.

1. Introduction

Neural networks are important nonlinear dynamical systems that simulate the structure and information transmission mechanism of biological neurons. With strong nonlinear approximation, self-learning, and parallel processing capabilities, neural networks have been widely used in pattern recognition, signal processing, intelligent robots, industrial automation, and other fields. However, in actual circuit implementation and network communication, due to the limited switching speed of amplifiers, signal transmission distance, and communication bandwidth constraints, time delay widely exists in neural networks.

Time delay is one of the important factors that affect the stability of neural networks. It will lead to performance degradation, oscillation, divergence, and chaos. In addition, in practical applications, limited by sensor cost, installation conditions, and measurement constraints, the output of the system usually contains only part of the state information, and it is difficult to obtain the full state directly. At the same time, external noise and disturbance will further affect the observation accuracy. Therefore, it is necessary to design an effective state estimator to realize the reconstruction of the unknown state.

State estimation has become a basic research direction of time-delay neural networks. Its core goal is to design an observer or estimator such that the error between the estimated state and the real state converges asymptotically. Since the beginning of the 21st century, scholars at home and abroad have carried out a lot of research on constant delay, time-varying delay, distributed delay, and random delay neural networks, and proposed a variety of robust state estimation methods.

With the development of time-delay system theory, the research of state estimation has experienced several stages: from simple Lyapunov function to complex Lyapunov-Krasovskii functional; from Jensen inequality to high-precision Wirtinger inequality and Bessel-Legendre inequality; from passive state estimation to H_∞, L_2-L_∞, and dissipative state estimation with multiple performance indicators; from time-triggered to event-triggered network-based state estimation.

This paper aims to provide a comprehensive and systematic review of state estimation for time-delay neural networks. It will help researchers quickly grasp the development context, mainstream methods, typical achievements, and future trends in this field.

2. Basic Model and Problem Description

2.1. Typical Time-Delay Neural Network Model

The most widely used continuous-time state-space model is described as:

$$\begin{gathered}\dot{x}(t)=-Ax(t)+W_1f(x(t))+W_2f(x(t-d(t)))+J+B_1\omega(t)\\ y(t)=Cx(t)+Dx(t-d(t))+B_2\omega(t)\\ z(t)=L_0x(t)+L_1x(t-d(t))\end{gathered}$$

Where $x(t)$ is the neuron state vector, $y(t)$ is the measurable output, $z(t)$ is the estimated signal, $d(t)$ is the time-varying delay, $f(\cdot)$ is the activation function, and $\omega(t)$ is the external disturbance.

2.2. Common Types of Time Delay

Common delay types include: Constant time delay, Time-varying delay, Interval time-varying delay, Distributed delay, Leakage delay, Markov jump delay, and Random delay. Different delay characteristics lead to differences in system dynamic behaviors and bring different challenges to state estimation design.

2.3. Activation Function Constraints

Activation functions usually meet the sector-bounded condition:

$$0 \leq \frac{f_j(s_1)-f_j(s_2)}{s_1-s_2} \leq l_j$$

This condition is the basis for using S-Procedure and stability analysis, which can effectively constrain the nonlinear characteristics of neural networks and simplify the design of state estimators.

3. Core Theoretical Tools and Analysis Methods

3.1. Lyapunov-Krasovskii Functional Method

The L-K functional is the most important tool for analyzing the stability of time-delay systems. By constructing a positive definite functional and proving its derivative is negative definite, the asymptotic stability or exponential stability of the system can be obtained. In recent years, the construction of L-K functional has developed towards delay-product-type functional, augmented vector functional, switched L-K functional, and functional with relaxed positive definiteness constraints, which greatly reduce the conservatism of stability criteria.

3.2. Integral Inequality Technology

Integral inequalities are used to estimate the upper bound of integral terms and reduce conservatism. Common ones include Jensen integral inequality, Park inequality, Wirtinger-based inequality, Bessel-Legendre inequality, Free-matrix-based inequality and Auxiliary-function-based inequality. Higher precision inequalities can significantly reduce the conservatism of stability criteria and improve estimation accuracy.

3.3. Reciprocally Convex Inequality

Reciprocally convex inequality is widely used to deal with time-varying delay terms. It can avoid excessive amplification of inequalities and improve the accuracy of estimation, which is an essential method for analyzing time-varying delay neural network systems.

3.4. Switched System Method

By dividing the delay interval and the sign of delay derivative, the original system is transformed into a switched system with multiple modes. Then mode-dependent estimators and switched L-K functionals are designed to fully use delay information, which solves the problem of insufficient utilization of time-delay information in traditional methods.

3.5. S-Procedure

S-Procedure is used to deal with the sector constraint of activation functions. Time-varying S-Procedure further relaxes matrix constraints and reduces conservatism, providing a more flexible theoretical basis for estimator design.

3.6. Linear Matrix Inequality

Almost all modern stability criteria and estimator design conditions are ultimately transformed into linear matrix inequality (LMI) problems, which can be solved efficiently by MATLAB LMI toolbox, realizing the parametric design of state estimators.

4. Research Progress of State Estimation

4.1. H_∞ State Estimation

H_∞ state estimation requires the system to be stable and suppress the influence of external interference within a certain level. It is one of the most mature robust estimation methods. The main development directions include delay-dependent and delay-derivative-dependent criteria, improved L-K functional, high-precision integral inequality, and switched model-based $H\infty$ estimation, which are widely used in anti-interference state estimation of neural networks.

4.2. L_2-L_∞ State Estimation

L_2-L_∞ state estimation focuses on minimizing the peak error under energy-bounded interference. It does not need noise statistical characteristics and has strong practicability. In recent years, improved Bessel-Legendre inequality and time-varying S-Procedure have been used to obtain lower conservative results, which further improve the transient performance of state estimation.

4.3. Dissipative State Estimation

Dissipative theory describes the energy gain and dissipation of the system. Generalized dissipativity can unify H_∞ performance, L_2-L_∞ performance, passivity performance and (Q, S, R)-dissipativity. It provides a unified framework for multi-objective estimator design and has become a hot research direction in recent years.

4.4. Event-Triggered State Estimation

In networked transmission, event-triggered mechanism only transmits data when necessary, which can greatly reduce communication pressure. The main developments include static event-triggered, adaptive event-triggered, memory-based event-triggered, and event-triggered combined with dissipativity. This method effectively solves the problem of limited network bandwidth in industrial networked control systems.

5. Existing Problems and Challenges

Firstly, the stability criteria still have certain conservatism, and the full utilization of time-delay quadratic information and dynamic characteristics needs to be further improved. Secondly, the contradiction between estimation accuracy and computational complexity is prominent, and high-precision algorithms often bring large computational overhead, which is not conducive to practical engineering application. Thirdly, the research on complex delay systems such as coupled delay and stochastic delay is insufficient. Fourthly, network security, cyber-attacks, and packet loss are rarely considered in networked state estimation. Finally, the practical application of relevant theoretical results in industrial scenarios needs to be further expanded.

6. Future Research Directions

Future research can be carried out from the following aspects: design of ultra-low conservative stability criteria based on novel integral inequalities and improved L-K functionals; intelligent state estimation combined with deep learning and machine learning algorithms; distributed state estimation under multi-sensor networks; secure state estimation resisting network cyber-attacks; low-consumption adaptive event-triggered estimation with lower communication rate; and engineering application verification in robots, unmanned aerial vehicles, smart grids and other intelligent equipment.

7. Conclusion

State estimation for time-delay neural networks is an important interdisciplinary direction involving control science, neural dynamics, and intelligent systems. With the development of high-precision inequalities, switched systems, and event-triggered mechanisms, the estimation performance of time-delay neural networks has been significantly improved, and the conservatism of traditional stability criteria has been effectively reduced. In the future, the research of state estimation will continue to develop towards lower conservatism, higher intelligence, stronger security, and wider engineering practicability, providing solid theoretical support for the intelligent application of neural network systems.

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