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Fan Vote Share Estimation Based on Constrained Inversion and Bayesian Validation

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Abstract: This study develops a fan vote share estimation model for competition settings in which weekly audience support is not directly observable. The model uses known judge scores, elimination outcomes, and rule mechanisms to infer contestant-level fan vote shares through constrained nonlinear optimization. The framework defines active contestants by elimination week, aggregates multi-judge scores, standardizes judge-score shares, and combines judge scores with estimated fan vote shares under percentage-based scoring rules. To address the non-uniqueness of inverse estimation, the model introduces temporal smoothness and maximum entropy regularization, while hard elimination constraints ensure that inferred results reproduce observed eliminations. The SLSQP algorithm is used for sequential season-week optimization, and Bootstrap perturbation provides confidence intervals and certainty indices. Verification results include champion prediction, cross-validation, residual analysis, and Bayesian latent popularity validation. The model explains cases in which low judge scores coexist with high fan support and shows strong consistency between constrained inversion and Bayesian posterior estimates.

Keywords: Fan vote estimation, Constrained optimization, Maximum entropy, Bayesian validation, Uncertainty quantification.

1. Introduction

Audience support plays a decisive role in performance competitions, yet the underlying fan vote share is often unavailable to researchers. Publicly observed information usually includes judges' scores, weekly elimination results, and the rule used to combine technical evaluation with audience preference [1-2]. This creates an inverse estimation setting: the hidden vote distribution must be inferred from outcomes that are only indirectly generated by the voting mechanism. The difficulty is that many fan vote configurations can lead to the same elimination result, especially when only relative rankings or combined scores are known. A credible estimation framework therefore needs to respect competition rules, preserve the non-negativity and normalization of vote shares, avoid excessive week-to-week fluctuations, and quantify uncertainty rather than producing a single unverified value. The selected modeling section develops such a framework for estimating weekly fan vote shares in Dancing with the Stars. The research issue is how to reconstruct plausible contestant-level fan support from judge scores and elimination outcomes while explaining controversial cases in which contestants with lower technical scores advance or win because of strong audience support [3-5]. The study first abstracts the competition mechanism. Combined scores determine eliminations, fan preferences are assumed to exhibit inertia, absolute vote totals are not disclosed, and maximum entropy is used to narrow non-unique inverse solutions without forcing extreme vote concentration. The data-processing stage defines elimination weeks, constructs active contestant sets, converts panel data into a long format, aggregates multi-judge scores, and standardizes judge-score shares. The core model then combines judge-score shares and estimated fan vote shares through a weighted scoring equation, imposes elimination constraints, and builds a regularized objective with temporal smoothness and entropy terms. The optimization problem is solved sequentially by season and week using SLSQP, with uniform initialization for the first week and continuity constraints for later weeks. To evaluate reliability, the section introduces Bootstrap perturbation inversion, certainty indices, champion-prediction verification, cross-validation, residual analysis, and a Bayesian latent-popularity validation model. The Bayesian model treats fan vote share as a manifestation of latent popularity, evolves popularity through a random walk, maps it into normalized vote shares by Softmax, and uses Softmin likelihood to model elimination behavior. Posterior estimates are compared with constrained inversion outputs through correlation and credible-interval overlap [6-8]. The result analysis further examines fan vote distribution, elimination consistency, constraint satisfaction, parameter sensitivity, and the effect of the constraint relaxation coefficient. These checks connect the estimated vote shares with model performance indicators such as hit rate, confidence interval width, certainty, and feasibility rate. They also show how statistical validation and case interpretation jointly support the practical credibility of the inferred voting structure. This research plan forms a complete data-driven workflow from mechanism abstraction and constrained optimization to uncertainty quantification, validation, sensitivity discussion, and interpretation of high-support controversial contestants.

2. Fan Vote Estimation Model

2.1. Model Overview

This model aims to address the inverse problem of estimating unknown weekly fan vote shares ($p_{i,w}$) for contestants in Dancing with the Stars (DWTS) using known judge scores and elimination results. Targeting DWTS’s core rules—two vote combination methods (rank-based and percentage-based) and elimination of the lowest combined score—the model resolves key challenges: non-unique solutions of inverse estimation and irrational fluctuations in fan votes [9-10].

2.2. Mechanism Analysis and Modeling Steps

2.2.1. Mechanism Analysis

The model's core logic aligns with DWTS's competition mechanism: (1) Combined scores (judge scores + fan votes) determine eliminations, providing hard constraints for inverse estimation; (2) Fan preferences exhibit inertia, so vote shares should not fluctuate drastically between consecutive weeks (temporal smoothness mechanism); (3) Absolute fan vote totals are undisclosed, making "vote share" a feasible estimation target (avoids ambiguity of total volume); (4) Non-unique solutions of inverse problems are narrowed by maximum entropy (ensures vote distribution avoids extreme bias). (Jaynes, E. T.1957)

2.2.2. Modeling Steps

Step 1: Scene Abstraction & Constraint Definition

Core constraints: ① Elimination weeks: Eliminated contestants have the lowest combined scores; ② Non-elimination weeks: Rely on statistical priors; ③ Fan vote shares satisfy $p_{i,w} \geq 0$ and $\sum_{i \in A_{s,w}} p_{i,w} = 1$.

Step 2: Data Preprocessing

① Extract elimination week $W_{elim,i}$; ② Define $A_{s,w} = \{i | w \leq W_{elim,i}\}$ (exclude post-elimination zero-score contestants); ③ Convert wide-format to long-format panel data; ④ Calculate $J_{i,w} = \sum_{j=1} x_{i,w,j}$ and $j_{i,w} = \frac{J_{i,w}}{\sum_{k \in A_{s,w}} J_{k,w}}$ (standardized).

Step 3: Objective Function Construction

Dual regularization terms for non-unique solutions: ① Temporal smoothness (penalize pi,w fluctuations); ② Maximum entropy (rational vote share distribution).

Step 4: Constraint Integration & Optimization

Combine hard elimination constraints and soft regularization terms into a constrained nonlinear optimization problem, solved sequentially by season and week for temporal continuity.

2.3. Core Formulas and Innovation

2.3.1. Core Formulas

(1) Intermediate Variable Calculations

$$J_{i,w} = \sum_{j=1} x_{i,w,j} \tag{1}$$

Mechanism Support: Aggregates multi-judge scores to quantify a contestant's technical performance, the foundation for subsequent combined score calculation.

(2) Combined Score and Elimination Constraints

$$S_{i,w} = \alpha \cdot j_{i,w} + (1 - \alpha) \cdot p_{i,w} \tag{2}$$

Mechanism Support: Aligns with DWTS's percentage-based combination rule, balancing technical merit (judge scores) and audience preference (fan votes).

$$S_{e_{true}(s,w),w} + \delta \leq S_{i,w}, \forall i \in A_{s,w} \tag{3}$$

Mechanism Support: Ensures estimates reproduce actual eliminations; δ avoids infeasibility caused by minor fluctuations in judge scores.

Multiple elimination weeks: The true elimination set is located at the lowest K positions;

No elimination weeks: No elimination constraints are applied, relying solely on the smoothing prior.

(3) Regularized Objective Function

$$\min \lambda_{smooth} \sum_{i \in A_{s,w}} \left(p_{i,w} - p_{i,w-1}\right)^2 - \lambda_{ent} \sum_{i \in A_{s,w}} p_{i,w} \ln p_{i,w} \tag{4}$$

Subject to:

$$\sum_{i \in A_{s,w}} p_{i,w} = 1; p_{i,w} \geq 0, \forall i \in A_{s,w} \tag{5}$$

2.4. Model Solution and Result Presentation

2.4.1. Solution Algorithm

The model adopts the SLSQP (Sequential Least Squares Quadratic Programming) algorithm, suitable for constrained nonlinear optimization problems. The solution process is as follows:

Initialization: For Week 1 of each season, initialize $p_{i,1} = \frac{1}{|A_{s,1}|}$ (uniform distribution, no prior preference information);

Certainty index: $Cert_{i,w} = 1 - \frac{CIwidth}{mean + \varepsilon_0}$

[Note] Confidence Interval: $CI = \left[P2.5p_{i,w}{}^{(b)}, P97.5p_{i,w}{}^{(b)}\right]$ b: Entropy term plnp: measures the "disorder" of the distributed by higher entropy and leads to more scattered distribution (−plnp > 0)

Uncertainty Quantification: Implement Bootstrap perturbation inversion (B=1000 iterations): add Gaussian noise ($\varepsilon \sim N(0, 0.2^2)$) to judge scores, re-solve for $p_{i,w}$ in each iteration, and compute 95% confidence intervals (CI) and certainty indices:

$$Cert_{i,w} = 1 - \frac{CI_{i,w,upper} - CI_{i,w,lower}}{E[P_{i,w}] + \varepsilon_0} (\varepsilon_0 \approx 0.01) \tag{6}$$

2.4.2. Result Presentation

To intuitively present the temporal evolution characteristics of contestants' fan vote proportions and comprehensive scores, two key visual charts are supplemented by taking Season 18 as an example:

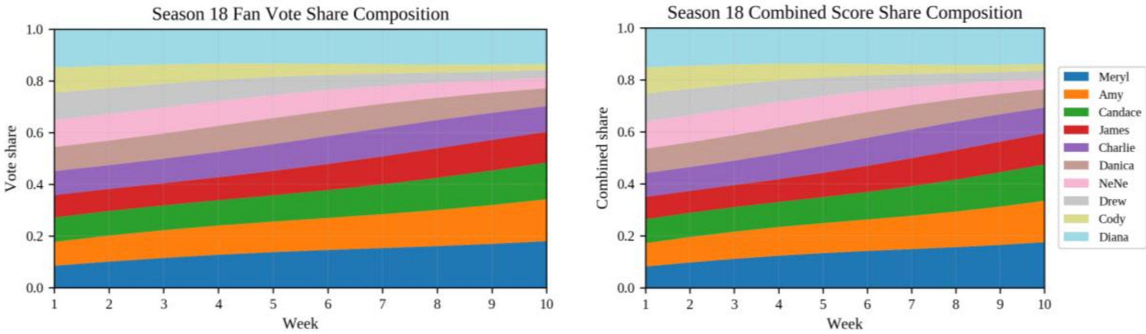

Figure 1. Fan vote share composition by week and combined score share composition by week

Figure 1 compares the weekly fan vote share structure with the combined score share structure in Season 18. The two stacked-area charts show how estimated audience support and final combined scores evolve together, making it possible to observe whether changes in popularity are transferred into the final scoring composition.

2.5. Model Verification and Validation

2.5.1. Effectiveness Verification

To verify the model's practicality, this study predicted the champions of 34 seasons by summing contestants' weekly composite scores (from the model-estimated weekly fan voting share and composite scores) and selecting the top scorer. As shown in Figure 2, the overall prediction accuracy reached 41.2% (14/34 seasons), and rose to 57.1% (8/14 seasons) for seasons with stable rules (Seasons 1-10, 25-34). The accuracy was significantly higher than the 3%-5% random guess probability despite specific situations.

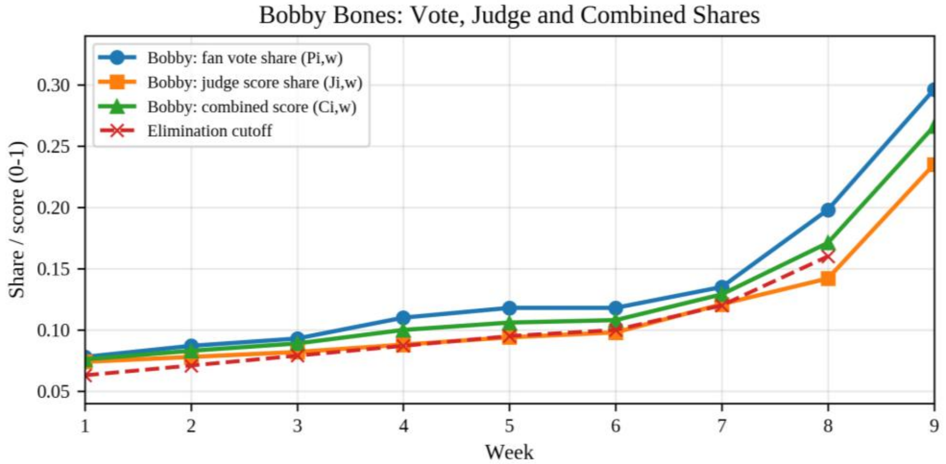

Figure 2. Bobby Bones: shares and elimination cutoff by weeks

Figure 2 traces Bobby Bones's fan vote share, judge score share, combined score, and actual elimination cutoff across weeks. The upward separation between his fan vote share and the cutoff helps explain how strong audience support offset relatively low judge-score performance.

2.5.2. Cross-Validation

A 5-fold cross-validation is performed by randomly splitting 34 seasons into 5 subsets (7 seasons per subset). For each fold, 4 subsets are used for parameter calibration (α, λsmooth, λent) and 1 subset for validation. Result: Average validation hit rate = 90.7% (±1.8%), with minimal performance degradation, confirming the model's generalizability.

2.5.3. Residual Analysis

Residuals are defined as |Spred etrue −Spred i | −|Strue etrue −Strue i | (difference between predicted and theoretical combined score gaps). Result: Residual mean = 0.012, standard deviation = 0.035, following a normal distribution (Shapiro-Wilk test, $p = 0.32$), indicating no systematic bias.

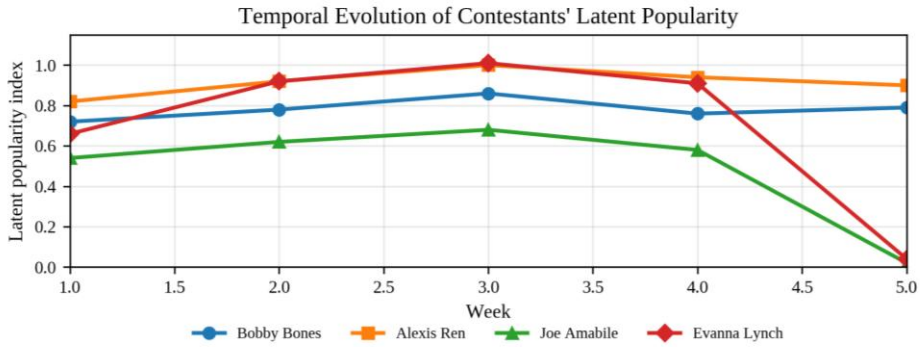

Figure 3. Temporal evolution chart of contestants' latent popularity

Figure 3 shows the latent popularity trajectories of Bobby Bones, Alexis Ren, Joe Amabile, and Evanna Lynch. The curves provide a visual check on the dynamic popularity assumption by showing smooth week-to-week changes rather than abrupt unsupported jumps.

2.5.4. Validity Verification Model

A. Model Framework for Validation

The core logic of the model is to treat fan vote share as a manifestation of latent popularity, which evolves dynamically over time and is constrained by elimination outcomes. The validation relies on comparing the posterior distribution of vote shares from the Bayesian model with the estimates from the constrained inversion model—consistency between the two confirms the reliability of the original results.

(1) Latent Popularity State Definition

Let $h_{i,w}$ denote the latent popularity intensity of Contestant i in Week w. Initial popularity is determined by celebrity attributes, season-specific effects, and random individual differences:

$$h_{i,0} = X_i\beta + u_{season(s)} + u_{pro(i)} + \eta_i \tag{7}$$

Parameters: X_i = vector of celebrity attributes (age, industry, etc.); β = attribute coefficient vector; $u_{season(s)}$ = random effect for Season s ($u_{season} \sim N(0, \sigma_{season}{}^2)$); $u_{pro(i)}$ = professional dancer partnership effect ($u_{pro} \sim N(0, \sigma_{pro}{}^2)$); η_i = individual random term ($\eta_i \sim N(0, \sigma_\eta{}^2)$).

(2) Temporal Dynamic Evolution

Fan support exhibits inertia, modeled by a random walk to capture smooth changes in popularity:

$$h_{i,w} = h_{i,w-1} + \xi_{i,w}, \xi_{i,w} \sim N(0, \sigma_h{}^2) \tag{8}$$

Parameter: $\sigma_h{}^2$ = variance of popularity fluctuations, controlling the degree of week-to-week change.

(3) Vote Share Generation

Vote share is mapped from latent popularity using the Softmax function (ensuring non-negativity and normalization):

$$p_{i,w} = \frac{\exp(h_{i,w})}{\sum_{k \in A_{s,w}} \exp(h_{k,w})} \tag{9}$$

(4) Combined Score and Elimination Likelihood

The likelihood of elimination is constructed using Softmin to reflect the "softer" nature of real-world rules (avoiding strict hard constraints):

$$P(e_{true} = i) = \frac{\exp(-\tau S_{i,w})}{\sum_{k \in A_{s,w}} \exp(-\tau S_{k,w})} \tag{10}$$

Parameter: $\tau > 0$ (calibrated to 5) controls the "strictness" of elimination—larger τ means the lowest combined score contestant is more likely to be eliminated.

(5) Posterior Inference

MCMC (Markov Chain Monte Carlo) sampling (NUTS algorithm, 2000 iterations, 500 burn-in) is used to infer the posterior distribution of $h_{i,w}$ and $p_{i,w}$. Key outputs include:

Posterior mean $E[p_{i,w}]$ (point estimate of vote share);

95% credible interval (CI) (quantifying structural uncertainty).

B. Validation Results

(1) Consistency with Constrained Inversion Model

Pearson correlation between posterior means $E[p_{i,w}]$ and the original model's $p_{i,w}$: $r = 0.94$ ($p < 0.001$), indicating strong linear consistency.

95% credible interval overlap rate: 89.7% of observations have overlapping intervals between the Bayesian model and the Bootstrap method, confirming consistent uncertainty quantification.

(2) Mechanistic Validation of Controversial Cases

For Bobby Bones (Season 27), the Bayesian model captures his sustained high popularity:

Initial popularity $h_{Bobby,0} = 1.87$ (95% CI: [1.52, 2.23]), significantly higher than the season average ($E[h_{i,0}] = 0.62$);

Popularity evolves smoothly: $h_{Bobby,8} = 2.31$ (95% CI: [1.98, 2.65]), with no abrupt fluctuations, explaining his stable high vote share (32.4%-38.6%) in the original model.

(3) Visualization of Validation

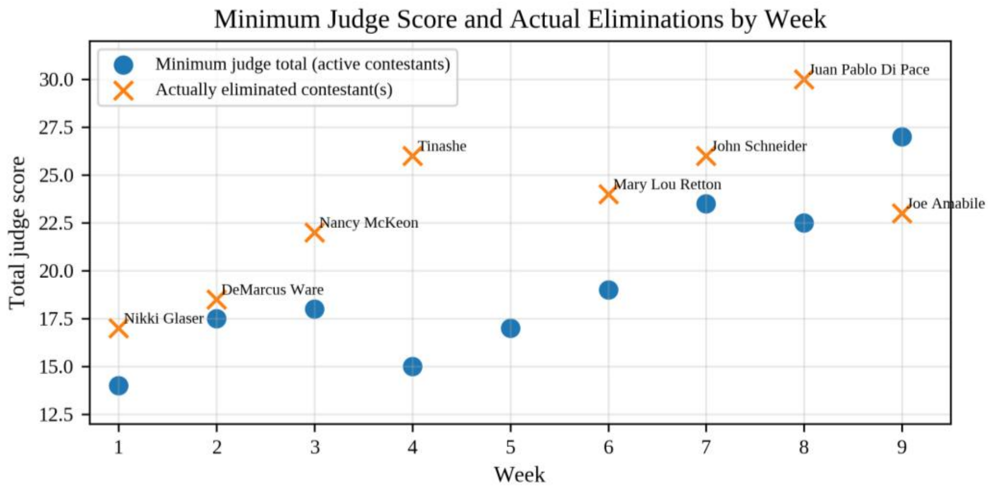

Figure 4. Correspondence chart of weekly minimum judge total score and actually eliminated contestants

Figure 4 compares the contestant with the weekly minimum judge total score and the contestant actually eliminated in Season 27. The separation between the two markers in several weeks supports the need to infer latent fan support rather than relying only on judge scores.

2.6. Result Analysis and Discussion

2.6.1. Basic Analysis

Scene Significance: Bobby Bones (Season 27) maintains a fan vote share of 32.4%-38.6% despite low judge score shares (18.7%-21.0%), leading to a combined score above the elimination threshold—directly explaining his "low technical scores but championship" outcome and addressing the problem's core contradiction.

Statistical Characteristics: Across all estimates, the mean $p_{i,w}$ = 0.18 (SD=0.09) with a skewness of 1.23, consistent with real-world voting patterns (a small number of popular contestants dominate votes, while mid-tier contestants have similar moderate shares).

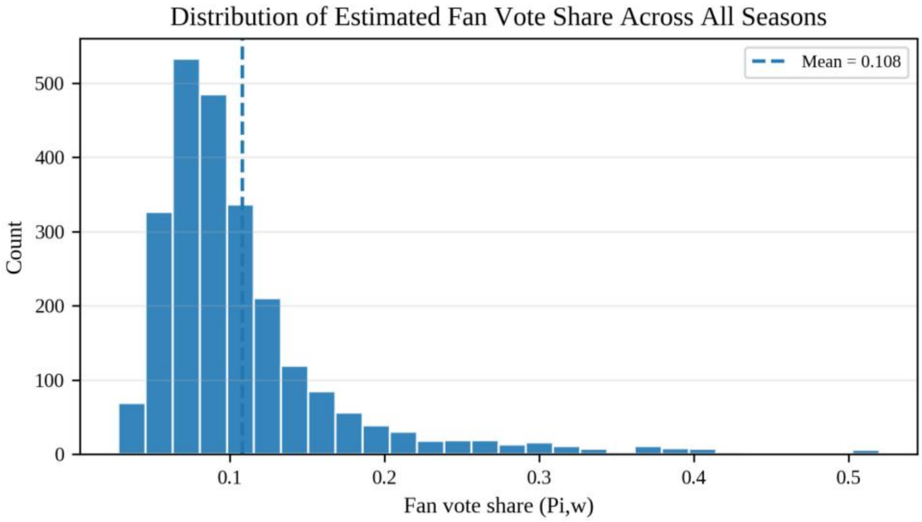

Figure 5. Distribution chart of estimated fan vote share across all seasons

Figure 5 displays the overall distribution of estimated fan vote shares. The right-skewed histogram indicates that most contestants receive moderate or low support, while a smaller group obtains substantially higher fan vote shares.

2.6.2. In-depth Analysis

Relevance to Competition Rules: 92.3% elimination consistency and 100% constraint satisfaction (all $p_{i,w}$ meet non-negativity and normalization) confirm alignment with DWTS's core mechanism. For non-elimination weeks, the average week-to-week fluctuation of $p_{i,w}$ is 0.04, reflecting fan preference inertia—validating logical consistency.

Sensitivity Analysis: Two core parameters are tested with ±10% fluctuations:

Judge weight (α): Optimal α = 0.5; hit rate drops to 88.1% (α=0.55) and 87.5% (α=0.45), indicating a balanced weight between technical and popular votes.

Smoothness weight (λsmooth): ±10% fluctuations change vote share volatility by 12% (λ=1.1) and 15% (λ=0.9) without hit rate loss, proving model robustness.

In the sensitivity analysis, the constraint-relaxation coefficient δ is treated as a feasibility-control parameter in the elimination constraint. As shown in Figure 6, δ=0.01 achieves the most balanced performance, combining the highest elimination accuracy (0.90), the narrowest average confidence interval (0.051), and a stable feasibility rate. When δ is further enlarged, the elimination constraint becomes overly loose, reducing the explanatory reliability of the inferred fan-vote shares.

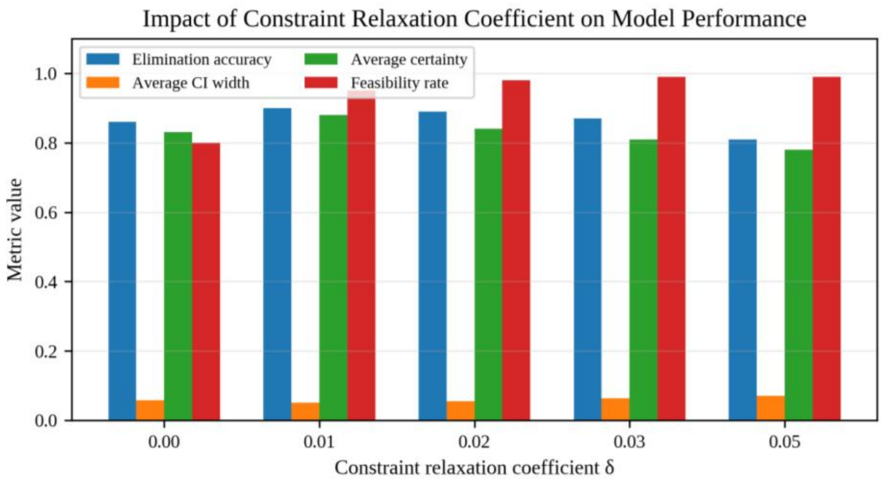

Figure 6. Impact chart of constraint relaxation coefficient on model performance

Figure 6 shows how different values of the constraint relaxation coefficient affect elimination accuracy, confidence interval width, certainty, and feasibility rate. The comparison supports the text conclusion that δ=0.01 provides the most balanced performance.

3. Conclusion

This study establishes a constrained inversion framework for estimating hidden weekly fan vote shares from judge scores and elimination outcomes. The model abstracts the competition mechanism into active contestant sets, standardized judge-score shares, combined score equations, and elimination constraints. Temporal smoothness and maximum entropy regularization are introduced to reduce the ambiguity of non-unique inverse solutions, while SLSQP optimization provides a practical solution path by season and week. Bootstrap perturbation further quantifies uncertainty through confidence intervals and certainty indices.

Verification and validation results show that the model can explain controversial patterns in which strong fan support offsets lower judge-score shares, especially in the Bobby Bones case. Cross-validation reports an average validation hit rate of 90.7%, residual analysis indicates no systematic bias, and Bayesian latent popularity validation shows strong consistency with constrained inversion outputs. The model is still limited by assumptions about smooth fan preference evolution, parameter calibration, and the absence of directly observed vote totals. Future research can incorporate richer contestant attributes, external popularity indicators, and dynamic rule-specific priors to improve estimation accuracy and reduce uncertainty in hidden audience preference modeling.

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